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# Full-Car Active Suspension Based on $H_2$ / Generalized $H_2$ Output Feedback Control

Shuyou Yu, Zhijie Wu, Fei Wang, Hong Chen

***Abstract*— This paper establishes a 15 degree-of-freedom full-car dynamic model for the car of Hongqi HQ3. Output feedback active suspension is designed based on a half-car model. Good ride comfort is achieved by minimizing the $H_2$ norm from disturbances of the road to the vertical acceleration and the pitch acceleration of the car, while safety constraints such as normalized suspension dynamic travels, static/dynamic load ratios and normalized actuator pressures are guaranteed by generalized $H_2$ norm. The 15 degree-of-freedom full-car model and left and right controllers are formed a closed-loop system. The simulation result of the full car shows that the active suspension can greatly reduce the vertical acceleration, pitch acceleration and roll acceleration as well as satisfy all constraints.**

## I. Introduction

The suspension system which is responsible for passing all the force and torque from the wheels to the sprung mass is an essential element of the vehicle. It is required to isolate the vibration of the vehicle from road disturbances for good ride comfort, and maintain continuous contact between the tyres and the road for good handling stability. In addition, the suspension strokes must stay within an allowable range. But it is hard to achieve a satisfactory compromise between ride comfort and stability using the traditional passive suspension system as the characteristics of springs and dampers are not controllable [1]. In contract, the active suspension may solve this problem effectively. The active suspension is formed by a passive suspension and a controllable actuator. So the characteristics of active suspension can change according to the road. In paper [2], an active vehicle suspension system based on preview control strategy is designed. But the effect of this active suspension is not very good when the velocity is high. In papers [3–6, 15], an active suspension system based on the state feedback constrained $H_\infty$ controller is designed. However, in general, not all the state variables of the active suspension are available. Predictive control of nonlinear active suspension is discussed in [7], where an explicitly analytical form of the optimal controller is given, online optimization is not required. However, it fails to deal with constraints.

Among the suspension specifications, only the ride comfort needs to be optimized, and others merely need to stay in some specified sets. So the active suspension control problem can be considered as an disturbances attenuation problem of constrained system. The $H_2$ control strategy is more propitious to reduce the disturbance of the random road and convex hull road effectively, and generalized $H_2$ norm is appropriate to describe the constraints of suspension systems [8][9]. Therefore, a multi-objective control algorithm — $H_2$ / generalized $H_2$ output feedback control is selected as the control strategy of the active suspension system in this paper. Furthermore, a vehicle dynamic model is used as the plant of the simulation. But generally the plant model is the same with the controller model in most papers about the active suspension, such as 2 degree-of-freedom quarter-car model, 4 degree-of-freedom half-car model [2][3][4]. These models can not reflect the vehicle dynamic characters exactly. In order to verify the effectiveness of the proposed active suspension system, a 15 degree-of-freedom full-car dynamic model is established in the multi-body dynamics software — AMESim (Advanced modeling environment for simulation of engineering systems).

This paper is organized as follows. In section II, we design a 15 degree-of-freedom full-car dynamic model in AMESim software. In section III, firstly, a four DOF half-car model and control problem are discussed. Then, $H_2$ / generalized $H_2$ output feedback control and controller design of half-car active suspension is exploited, respectively. In Section IV, a simulation test is implemented where the vehicle passes a road with convex hull at a uniform speed. A short summary is given in Section V.

## II. full-car dynamics model

Full-car modeling based on traditional mathematical methods is a complex process, and it's difficult to guarantee the accuracy of the model. The development of multi-body dynamics and the related software provide a powerful solution to the problem. In this paper, multi-body dynamics software AMESim is selected as the tool for full-car modeling and simulation. Furthermore, a 15 degree-of-freedom full-car model is established in the AMESim software. The model is used as the plant of active suspension system. The construction of the vehicle model is displayed in Fig 1. We can see that the vehicle model is composed of several sub-systems, such as chassis, suspension, steering system, driveline and brake system, seat system, road, tyre, etc. The parameters of vehicle model is from the car of Hongqi HQ3 which is produced by the First Automobile Works (FAW) in China.

The chassis is a multi-body system, which contains vehicle body, steering gear, axles, wheels and the connecters between them. There are 15 degree-of-freedom in the chassis system

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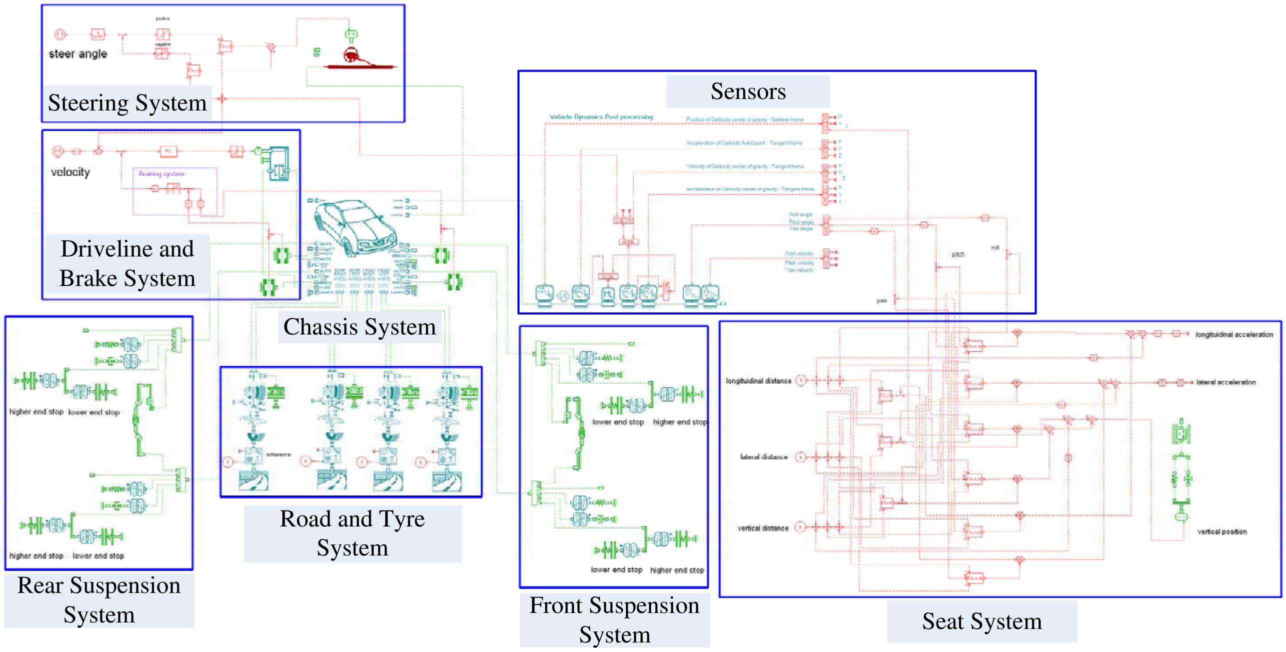



Fig. 1. Vehicle model based on AMESim

model: vehicle body – 6, steering gear – 1, axles – 4, wheels – 4. Chassis system model is the key part of the vehicle model, and all the other submodels are connected with it. The full-car model is based on the chassis system model in this paper.

The suspension has a direct impact on the vehicle performance since it can isolate the vibration from the road to the car body. The suspension is composed of spring, damper, anti-roll bar and limit blocks [10]. The suspension stiffness and damping determine the vehicle's handling stability and ride comfort. The anti-roll bar can reduce the roll movement of the vehicle.

The tyre model used in this paper is the magic tire formula which has been widely used in the automotive field. The magic tire formula defines the longitudinal force $F_x$, the lateral force $F_y$ and the aligning torque $M_z$ with only one formula [11][12].

In order to evaluate the vehicle's ride comfort, a driver's seat is modeled. The car body can be approximated as a rigid body. The seat is connected with the chassis through a shock absorber which is formed by a spring and a damper [13]. The relative displacement of the contact point between the chassis and the shock absorber $\Delta x_{seat}$, $\Delta y_{seat}$, $\Delta z_{seat}$ can be calculated

$$\begin{cases} \Delta x_{seat} = \sin(\arctan\frac{a}{h}-\theta)\sqrt{a^2+h^2} \\ \qquad\quad + \sin(\arctan\frac{a}{b}-\psi)\sqrt{a^2+b^2}-2a \\ \Delta y_{seat} = \sin(\arctan\frac{b}{h}+\varphi)\sqrt{b^2+h^2} \\ \qquad\quad + \cos(\arctan\frac{a}{b}-\psi)\sqrt{a^2+b^2}-2b \\ \Delta z_{seat} = 2h-\cos(\arctan\frac{a}{h}-\theta)\sqrt{a^2+h^2} \\ \qquad\quad - \cos(\arctan\frac{b}{h}+\varphi)\sqrt{b^2+h^2} \end{cases}$$

where $\theta$, $\psi$ and $\varphi$ represent the pitch angle, the yaw angle and the roll angle of the car body; $a$, $b$ and $h$ represent the distance between the contact point and the vehicle center of gravity in the $x$, $y$ and $z$ direction.

For the need of the simulation, we design a steering system, drive-line system and brake system. Through these systems the vehicle can track a given velocity and steering angle. The accuracy of the vehicle model is validated in term of the vehicle dynamics simulation with the step angle input of steering wheel specified by the national standard of China. The results of the simulation and the data of the real car Hongqi HQ3 are compared in the Fig 2 with the same velocity and steering wheel angle. The vehicle model can meet the requirements of the vehicle dynamics simulation experiments.

## III. Active Suspension Design

Here, we discuss the control of half car active suspensions based on $H_2$ / generalized $H_2$ output feedback control.

### A. $H_2$ / generalized $H_2$ output feedback control

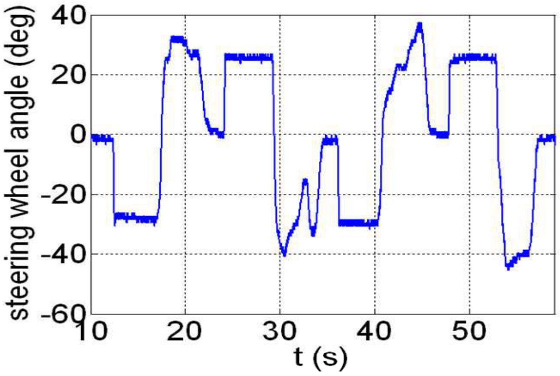
Consider linear time-invariant (LTI) systems

$$\begin{cases} \dot{x}(t) = Ax(t) + B_w w(t) + B_u u(t) \\ z_1(t) = C_1 x(t) + D_1 w(t) + D_{1u} u(t) \\ z_2(t) = C_2 x(t) + D_2 w(t) + D_{2u} u(t) \\ y(t) = C_y x(t) + D_{yw} w(t) \end{cases} \tag{1}$$

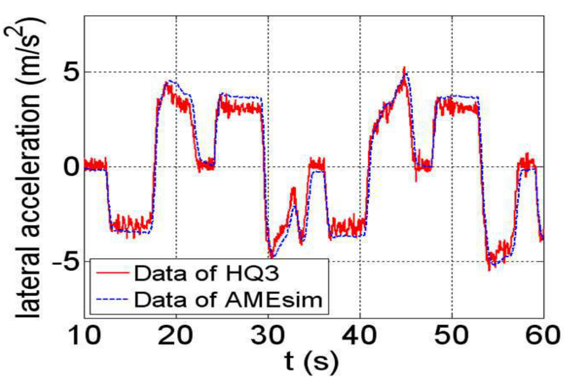
where $x \in R^{n_x}$ is the system state, $u \in R^{n_u}$ is the control inputs, $w \in R^{n_w}$ is exogenous inputs, $z_1 \in R^{n_{z1}}$ is the outputs related to the performance of the system, $z_2 \in R^{n_{z2}}$ is the outputs related to the constraint conditions of the system, and $y \in R^{n_y}$ is the feedback outputs.

Consider an output feedback controller

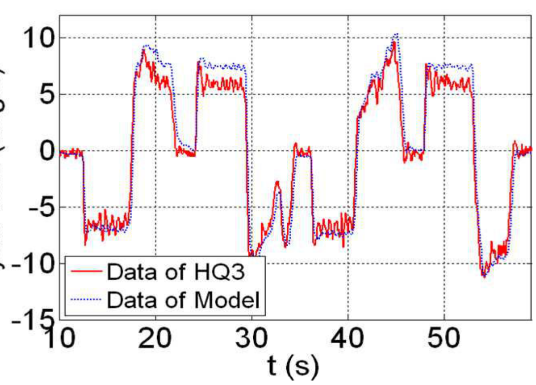
$$\begin{cases} \dot{\zeta}(t) = A_K\zeta(t) + B_K y(t) \\ u(t) = C_K\zeta(t) + D_K y(t) \end{cases} \tag{2}$$



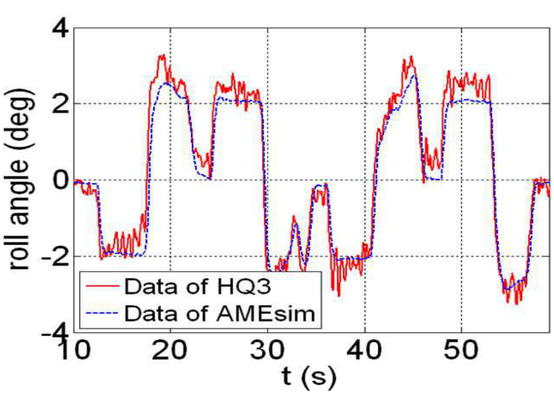

(a) Steering wheel angle




(b) Lateral acceleration




(c) Yaw rate




(d) Roll angle

Fig. 2. Result of the model validation

Denote $x_{cl} = \begin{bmatrix} x^T & \zeta^T \end{bmatrix}^T$, the closed-loop system can be rewritten as:

$$\left\{ \begin{array}{l} \dot{x}_{cl}(t) = A_{cl}x_{cl}(t) + B_{cl}w(t) \\ z_1(t) = C_{cl,1}x_{cl}(t) + D_{cl,1}w(t) \\ z_2(t) = C_{cl,2}x_{cl}(t) + D_{cl,2}w(t) \end{array} \right. \tag{3}$$

where

$$\begin{array}{l} A_{cl} = \begin{pmatrix} A + B_u D_K C_y & B_u C_K \\ B_K C_y & A_K \end{pmatrix} \\ B_{cl} = \begin{pmatrix} B_w + B_u D_K D_{yw} \\ B_K D_{yw} \end{pmatrix} \\ C_{cl,i} = \begin{pmatrix} C_i + D_{iu} D_K C_y & D_{iu} C_K \end{pmatrix}, \\ D_{cl,i} = D_i + D_{iu} D_K D_{yw}, \quad i = 1, 2 \end{array}$$

Let $G_1$ denote the closed-loop transfer function from $w$ to $z_1$, and $G_2$ denote the closed-loop transfer function from $w$ to $z_2$. $H_2$ / generalized $H_2$ output feedback control scheme is to find an output-feedback controller (2), which minimizes $||G_1||_2$ as well as $\|G_2\|_g \leq \rho$.

Suppose there is a matrix $P \in R^{2n_x \times 2n_x}$ with $P > 0$. Partition $P$ and $P^{-1}$ as

$$P = \begin{pmatrix} Y & N \\ N^T & \star \end{pmatrix}, \; P^{-1} = \begin{pmatrix} X & M \\ M^T & \star \end{pmatrix}$$

with $YX + NM^T = I$ and $XY + M^T N^T = I$, where $\star$ is an unknown appropriate matrix, and $X, Y, M, N \in R^{n_x \times n_x}$. Define new variables

$$\begin{array}{l} \hat{A} := NA_K M^T + NB_K C_y X + Y B_u C_K M^T \\ \qquad + Y(A + B_u D_K C_y)X \\ \hat{B} := NB_K + Y B_u D_K \\ \hat{C} := C_K M^T + D_K C_y X \\ \hat{D} := D_K \end{array}$$

An output feedback controller can be obtained by solving the following optimization problem [15][16]

$$\min_{\nu, S, X, Y, \hat{A}, \hat{B}, \hat{C}, \hat{D}} \nu^2 \quad , \quad s.t. \;\; (5) \tag{4}$$

$$\begin{array}{l} \begin{pmatrix} \mathfrak{A}_{11} & \mathfrak{A}_{12} & \mathfrak{A}_{13} \\ \circlearrowright & \mathfrak{A}_{22} & \mathfrak{A}_{23} \\ \circlearrowright & \circlearrowright & -I \end{pmatrix} < 0 \\ \begin{pmatrix} X & \circlearrowright & \circlearrowright \\ I & Y & \circlearrowright \\ C_1 X + D_{1u}\hat{C} & C_1 + D_{1u}\hat{D}C_y & S \end{pmatrix} > 0 \\ \begin{pmatrix} X & I & (C_2 X + D_{2u}\hat{C})^T \\ \circlearrowright & Y & (C_2 + D_{2u}\hat{D}C_y)^T \\ \circlearrowright & \circlearrowright & \rho^2 I \end{pmatrix} > 0 \\ D_1 + D_{1u}\hat{D}D_{yw} = 0 \\ D_2 + D_{2u}\hat{D}D_{yw} = 0 \\ Trace(S) < \nu^2 \end{array} \tag{5}$$

where $\circlearrowright$ denotes the symmetric part of a matrix and
$\mathfrak{A}_{11} = AX + XA^T + B_u\hat{C} + (B_u\hat{C})^T$
$\mathfrak{A}_{12} = \hat{A}^T + (A + B_u\hat{D}C_y)$
$\mathfrak{A}_{13} = B_w + B_u\hat{D}D_{yw}$
$\mathfrak{A}_{22} = A^T Y + YA + \hat{B}C_y + (\hat{B}C_y)^T$
$\mathfrak{A}_{23} = YB_w + \hat{B}D_{yw}$

If the optimization problem (4) has a solution ($\nu^*$, $S^*$, $X^*$, $Y^*$, $\hat{A}^*$, $\hat{B}^*$, $\hat{C}^*$, $\hat{D}^*$), and there exists nonsingular matrixes $M$ and $N$ such that $MN^T = I - X^*Y^*$, then the output feedback control (2) can be define

$$\begin{cases} D_K := \hat{D}^* \\ C_K := (\hat{C}^* - D_K C_y X^*)M^{-T} \\ B_K := N^{-1}(\hat{B}^* - Y^* B_u D_K) \\ A_K := N^{-1}(\hat{A}^* - N B_K C_y X^* - Y^* B_u C_K M^T \\ \qquad - Y^*(A + B_u D_K C_y)X^*)M^{-T} \end{cases} \tag{6}$$

## B. Four DOF Half-car Model and Problem Statement

Since the full-car model is symmetric, we decouple it into two half-car models, namely the bounce/pitch and roll/wrap half-cars [4, 20]. Fig 3 gives a schematic presentation of a half-car model. The linearized dynamic equation of the half-car vehicle dynamic equations are given by Equ. (7), which has 4 degree-of-freedom: the car body's vertical motion and pitch motion, the wheels' vertical motions [17, 18].

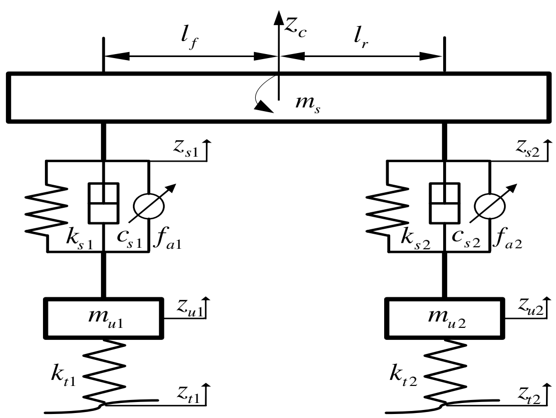



Fig. 3. Structure of the half-car model

The normalized weighting function of the road profile is defined [15]

$$\dot{z}_t = 2\pi n_0 \sqrt{G_0 V} w(t) \tag{8}$$

where $w(t)$ is a white noise whose mean value is 0 and power spectral density is 1, $n_0$ is the reference space frequency, $G_0$ is the road roughness coefficient, $V$ is the velocity of the vehicle.

The active force $f_{a1}$ and $f_{a2}$ are, in general, generated by hydraulic actuators placed between the sprung and the unsprung masses [4]. The model of the actuator is [19]

$$\begin{cases} f_a = A_r P_L \\ \frac{V_t}{4\beta_e}\dot{P}_L = Q_a - C_{tp}P_L - A_r(\dot{z}_s - \dot{z}_u) \end{cases} \tag{9}$$

where $f_a$ is the active force, $A_r$ is the actuator ram area, $P_L$ is the actual pressure, $Q_a$ is the flow rate, $\beta_e$ is the effective bulk modulus, $C_{tp}$ is the coefficient of total leakage due to pressure, $V_t$ is the total actuator volume.

The values of the half-vehicle model parameters used in the controller design were given in Table I. Both the sprung mass $m_s$ and the pitch inertia $I_\phi$ are half of the value of the full car vehicle.

TABLE I
HALF-VEHICLE DYNAMICS MODEL PARAMETERS

| parameters | values | parameters | values |
|---|---|---|---|
| $m_s$ | 742.8 $kg$ | $I_\phi$ | 1385 $kg{\cdot}\mathrm{m}^2$ |
| $m_{u1}$ | 46.6 $kg$ | $m_{u2}$ | 49.2 $kg$ |
| $l_f$ | 1.368 $m$ | $l_r$ | 1.482 $m$ |
| $c_{s1}$ | 3023 $N$·s/m | $c_{s2}$ | 2894 $N$·s/m |
| $k_{s1}$ | 27450 $N/m$ | $k_{s2}$ | 35550 $N/m$ |
| $k_{ti}$ | 249315 $N/m$ | $S_{\max}$ | 0.08 $m$ |
| $F_{\max}$ | 1500 $N$ | $A_r$ | 3.35 $\times 10^{-4} m^2$ |
| $Q_s$ | 2 $\times 10^{-4} m^3/s$ | $P_s$ | 1.03 $\times 10^7 Pa$ |
| $\frac{4\beta_e}{V_t}$ | 4.515 $\times 10^{13} N/m^5$ | $\alpha C_{tp}$ | 1 $s^{-1}$ |

## C. Controller Design

Here, the control law of the active suspension is designed based on a half-car model. In order to quantify ride comfort, the vertical acceleration and the pitch acceleration of the car are chosen as the performance output $z_1$.

Good handling requires a firm uninterrupted contact of wheels to road. That is, the dynamic tire load should not exceed the static ones [3, 4]

$$k_{ti}(z_{ui}(t) - z_{ri}(t)) \leq F_i, \quad i = 1, 2, \forall t \geq 0 \tag{10}$$

where the static tire loads $F_1$ and $F_2$ are computed as follows

$$F_1 = (l_r m_s g + (l_f + l_r)m_{u1}g)/(l_f + l_r)$$
$$F_2 = (l_f m_s g + (l_f + l_r)m_{u2}g)/(l_f + l_r)$$

and $g$ is the acceleration of gravity. Moreover, the suspension stroke limitation

$$|z_{si}(t) - z_{ui}(t)| \leq S_{max}, \quad i = 1, 2, \forall t \geq 0 \tag{11}$$

have to be taken into account to prevent excessive suspension bottom which will lead to rapid deteriorate of ride comfort and possible structural damage. Considering actuator saturation [4], the load flow is bounded

$$|Q_i(t)| \leq Q_s, \quad i = 1, 2, \forall t \geq 0, \tag{12}$$

where $Q_s$ is the rated load flow. Clearly, (10), (11) and (12) can be treated as time-domain hard constraints. Thus, $z_2$ contain normalized suspension dynamic travels, static/dynamic load ratios and normalized actuator pressures. Assume that vertical acceleration, suspension dynamic travels and normalized actuator pressures can be measured, then $y$ contains vertical acceleration, suspension dynamic travels and normalized actuator pressures.

Define $x = \begin{bmatrix} z_{s1} - z_{u1} & z_{s2} - z_{u2} & \dot{z}_{s1} & \dot{z}_{s2} & z_{u1} - z_{t1} & z_{u2} - z_{t2} & \dot{z}_{u1} & \dot{z}_{u2} & P_{L1}/P_s & P_{L2}/P_s \end{bmatrix}^T$ $u = \begin{bmatrix} \frac{Q_{a1}}{Q_s} & \frac{Q_{a2}}{Q_s} \end{bmatrix}^T, w = \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T$, where $P_s$ is the hydraulic supply pressure, then the active suspension system based on the half-car model can be written in the form of (1).

An output feedback control law is designed which minimizes $\|G_1\|_2$ as well as $\|G_2\|_g \leq 1$. That is, the ride comfort of the vehicle is improved and all the safety constraints are satisfied with respect to disturbances of the road.

$$
\begin{cases}
\ddot{z}_{s1} = -\left(\frac{k_{s1}}{m_s} + \frac{k_{s1} l_f^2}{I_\theta}\right)(z_{s1} - z_{u1}) - \left(\frac{k_{s2}}{m_s} - \frac{k_{s2} l_f l_r}{I_\theta}\right)(z_{s2} - z_{u2}) - \left(\frac{c_{s1}}{m_s} + \frac{c_{s1} l_f^2}{I_\theta}\right)(\dot{z}_{s1} - \dot{z}_{u1}) \\
\qquad - \left(\frac{c_{s2}}{m_s} - \frac{c_{s2} l_f l_r}{I_\theta}\right)(\dot{z}_{s2} - \dot{z}_{u2}) + f_{a1}\left(\frac{1}{m_s} + \frac{l_f^2}{I_\theta}\right) + f_{a2}\left(\frac{1}{m_s} - \frac{l_f l_r}{I_\theta}\right) \\
\ddot{z}_{s2} = -\left(\frac{k_{s1}}{m_s} - \frac{k_{s1} l_f l_r}{I_\theta}\right)(z_{s1} - z_{u1}) - \left(\frac{k_{s2}}{m_s} + \frac{k_{s2} l_r^2}{I_\theta}\right)(z_{s2} - z_{u2}) - \left(\frac{c_{s1}}{m_s} - \frac{c_{s1} l_f l_r}{I_\theta}\right)(\dot{z}_{s1} - \dot{z}_{u1}) \\
\qquad - \left(\frac{c_{s2}}{m_s} + \frac{c_{s2} l_r^2}{I_\theta}\right)(\dot{z}_{s2} - \dot{z}_{u2}) + f_{a1}\left(\frac{1}{m_s} - \frac{l_f l_r}{I_\theta}\right) + f_{a2}\left(\frac{1}{m_s} + \frac{l_r^2}{I_\theta}\right) \\
\ddot{z}_{u1} = \frac{k_{s1}}{m_{u1}}(z_{s1} - z_{u1}) + \frac{c_{s1}}{m_{u1}}(\dot{z}_{s1} - \dot{z}_{u1}) - \frac{k_{t1}}{m_{u1}}(z_{u1} - z_{r1}) - \frac{f_{a1}}{m_{u1}} \\
\ddot{z}_{u2} = \frac{k_{s2}}{m_{u2}}(z_{s2} - z_{u2}) + \frac{c_{s2}}{m_{u2}}(\dot{z}_{s2} - \dot{z}_{u2}) - \frac{k_{t2}}{m_{u2}}(z_{u2} - z_{r2}) - \frac{f_{a2}}{m_{u2}}
\end{cases}
\tag{7}
$$

## IV. Closed-Loop Testing

In this section, the active suspension based on the $H_2$ / generalized $H_2$ output feedback control strategy is connected with the 15 degree-of-freedom full-car model in AMESim software which form a closed-loop system. There are two active suspension controllers where the left controller is related to the left half-car, and the right controller is related to the right half-car. The feedback signals $(y_L, y_R)$ contain unilateral body vertical acceleration $(\ddot{z}_L, \ddot{z}_R)$, body pitch acceleration $\ddot{\theta}$, suspension dynamic travel $z_{sij} - z_{uij}(i, j = 1, 2)$ and actuator normalized pressure $P_{Lij}/P_s(i, j = 1, 2)$.

The vehicle dynamic simulation experiment is carried out to verify the effect of the robust controller. In the simulation, the vehicle left wheels pass a convex hull road with the speed of $30km/s$. The convex hull road can cause severe vibration of the vehicle which affects the vehicle handling stability and ride comfort seriously. The response curves of the passive suspension and the active suspension are shown in Fig 4. The car body vertical acceleration, the pitch acceleration, the roll acceleration and the seat vertical acceleration of the vehicle with active suspension are significantly reduced. Furthermore, the active forces, dynamic travels and static/dynamic load ratios of the active suspension do not exceed the boundaries.

## V. Conclusions

This paper presented a 15 degree-of-freedom full-car model based on AMESim software, which can be used as the plant for the vehicle active safety control. Then the $H_2$ / generalized $H_2$ output feedback control algorithm was introduced, and an active suspension system based on it was developed. Finally, a closed-loop system was established with the full-car model in AMESim, and a simulation experiment is implemented in which the vehicle passes a road with convex hull at a uniform speed. The simulation results showed that the proposed active suspension greatly improves the handling stability and ride comfort of the vehicle as well as constraints satisfaction.

## VI. Acknowledgement

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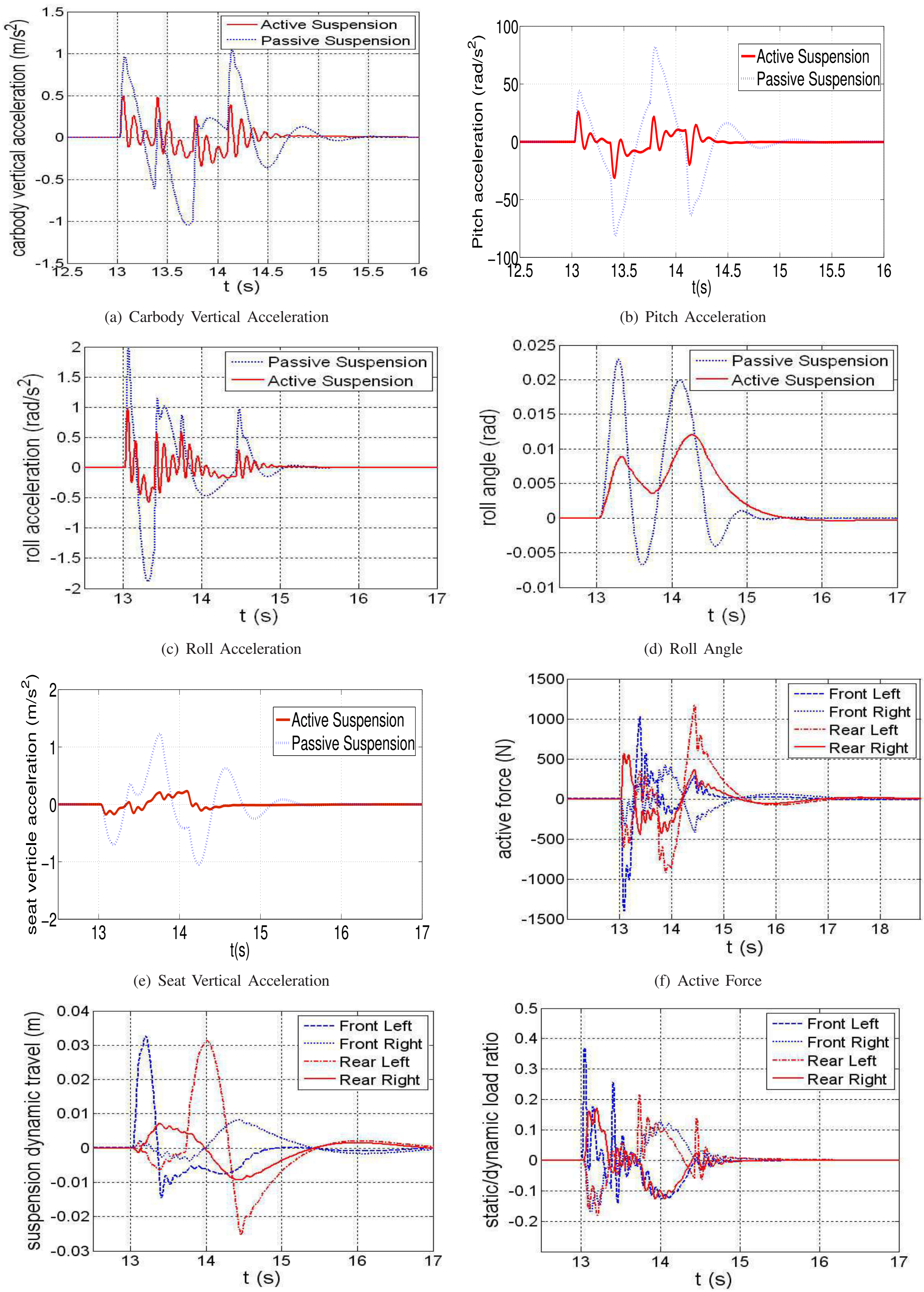


(a) Carbody Vertical Acceleration

(b) Pitch Acceleration

(c) Roll Acceleration

(d) Roll Angle

(e) Seat Vertical Acceleration

(f) Active Force

(g) Suspension Dynamic Travel

(h) Static/Dynamic Load Ratio

Fig. 4. Result of the Simulation